**The Fourteenth Colloquiumfest,**

**February 28 and March 1, 2014**

**University of Saskatchewan**

The Colloquiumfest is a small, (almost) annual low budget conference on algebra and logic that started in the year 2000 at the University of Saskatchewan with a meeting in honor of the 60th birthday of Murray Marshall. With its fifth meeting, it started traveling around the world. It was held at the IHP in Paris, the CUNY Graduate Center in New York, at Bogazici University in Istanbul, at Valladolid in Spain, in the Polish and in the Czech mountains. A list of the past Colloquiumfest meetings can be found at the Valuation Theory Home Page http://math.usask.ca/fvk/Valth.html.

The Eleventh Colloquiumfest in 2011 and this year's Fourteenth Colloquiumfest (both at the University of Saskatchewan) were generously supported by the Pacific Institute for the Mathematical Sciences. The latter meeting has expanded the original scope of the Colloquiumfest series. We explored the present state of Fixed Point Theory and its applications in analysis, algebra and computer science. The second theme was the Theory of Fractals, in particular the various existing definitions and the question how to adapt them to structures that appear to be fractal but do not match the classical definitions. Both subjects are interwoven and also have interesting connections with topology. We discussed new directions and research topics in these areas.

The Fourteenth Colloquiumfest was organized by Franz-Viktor Kuhlmann and J.C. Wang (Department of Mathematics and Statistics, UofS), Chris Dutchyn (Department of Computer Science, UofS), and Katarzyna Kuhlmann (Institute of Mathematics, University of Silesia at Katowice, Poland).

Our meeting started with a Department Colloquium of the Department of Computer Science. Robin Cockett of the University of Calgary reviewed the categorical foundations of sequential programming languages (where fixed-points correspond to datatype constructions) and extended this to linear categories that model message-passing. As a result, he motivated the observation that protocols in concurrent programming show up as fixed points of these specialized algebras.

After a well deserved coffee break in its beautiful lounge, the Department of Mathematics and Statistics held its Department Colloquium. Anthony To-Ming Lau of the University of Alberta discussed his recent work in fixed point and geometric properties for Fourier and Fourier-Stieltjes algebras which arise naturally in the harmonic analysis of locally compact groups. Through this talk, we have learned that many important geometric properties of these algebras can be characterized by the fixed point property in the dual spaces. Thus, the fixed point property at the dual level may serve as a good guideline for the study of these algebras. The talk concluded with several open questions in this field.

The idea of computable numbers is of foundational significance in computer science and has had a significant impact on logic. However, despite arguments and challenges noted by prominent researchers, most mathematical models, and hence most physical models (including models of quantum mechanics) depend on uncomputable numbers, that is, the continuum of real (or complex) numbers. This observation motivates a foundational study of quantum mechanics from a computational perspective. Amr Sabry of Indiana University at Bloomington introduced tha audience to his program to investigate variants of quantum mechanics formulated with computable number systems, analyze their computational power, and compare to the conventional model based on uncomputable numbers.

Algorithms are designed to obtain increasingly accurate representations of a desired number or object. About a quarter century ago, Steve Matthews invented a generalized metric that simultaneously models closeness and the partiality of knowledge. Ralph Kopperman of the City College of New York showed us that complete partial metric spaces have a contraction fixed point theorem that gives conditions under which an algorithm will approach a desired object arbitrarily closely. Partial metrics can also be generalized to yield all topologies.

In a second talk, Kopperman discussed an object which all of us face almost every day: the computer screen. It looks like a rectangle in the plane, and it would be helpful if we could get it to act like one topologically - for example, if it were a connected space, but curves separated regions. But finiteness leads to issues; for example, not all its points can be closed for if they were, then all its subsets would be both closed and open, so the space would be totally disconnected. Kopperman showed how we can deal with these problems, introducing the ideas of digital topology and finite approximation. He convinced us that once we become used to finite spaces, we see that we can usefully approximate the best topological spaces using them.

Several talks in our meeting addressed possible ideas for the generalizations of the notion of "fractal". One of the common definitions of "fractal" describes it as the fixed set of an Iterated Function System (IFS), in which the functions are contractions with respect to a usual metric. Wieslaw Kubis, who is presently at the Czech Academy of Sciences, discussed the notion of topological IFS, where the contractive property of functions is defined in topological terms. He showed that it is equivalent to a system of weak contractions with respect to some compatible metric.

Franklin Mendivil of Acadia University discussed the problem of generalizing the classical IFS definition to the setting of set-valued measures. This requires setting up a useful metric on a space of measures along with a natural fractal operator which is contractive on this space. Mendivil presented two different notions of set-valued measure and concluded with examples showing how fractal measures can be generalized to recursive partitioning schemes.

The ultrametric version of Banach's Fixed Point Theorem, first published by S. Priess-Crampe in 1990, differs crucially from the metric case in that the fixed point of a function is not necessarily obtained after a sequence of iterations indexed by the natural numbers. Transfinite induction has to be used. This can be called "nonarchimedean behavior". The question arises whether ITFs can make sense in nonarchimedean settings, for instance on nonarchimedean ordered fields (where the distance function does not have values in the reals anymore) or on valued fields. This question had been addressed already at a previous workshop organized by Kubis and the Kuhlmanns.

Tristan Tager, a Ph.D. student at Indiana University, Bloomington, explained to the audience how generalizing to nonarchimedean settings forces us to think about generalizations of the basic notions, such as "metric" and "compact space". He discussed examples, such as power series fields which show the limitations of the classical notion of IFS. One generalization that Tager studies uses an abstract notion of "radius" of balls around points.

Hyperspaces form a powerful tool in mathematics: lots of fractal and other geometric objects can be viewed as fixed points of functions in suitable hyperspaces - as well as interesting classes of formal languages in theoretical computer science. As we heard from Rene Bartsch of the TU Darmstadt in Germany, there are many connections between hyperspaces and function spaces in topology. Thus results from hyperspaces help to get new results in function spaces and vice versa. Unfortunately, there is no natural hyperspace construction known for general topological categories (in contrast to the situation for function spaces). Bartsch proposed a new approach to define hyperstructures which works in every cartesian closed topological category, and so applies to every topological category, using its topological universe hull.

The meeting had a sizable audience of faculty, many graduate and some undergraduate students from Mathematics & Statistics and Computer Science. Several of the students showed extensive interest in various topics and had fruitful discussions with the speakers. It was amazing to see the lievely interaction of participants from seemingly very different areas, finding common (fixed) points. As several of the speakers stayed for up to a week, we were able to follow-up on ideas from the talks in smaller groups. An informal seminar talk was added right before the start of the meeting: Franz-Viktor Kuhlmann reported on his joint work with Katarzyna Kuhlmann which presents a general framework for Fixed Point Theorems. It takes its intuition from the ultrametric world, centering the interest on "spherically complete ball spaces". In this approach, the notion of "radius" of a ball is discarded altogether.

The coffee breaks of our meeting were particularly important: while stirring our coffees we observed many fixed points, and discussed the promising relations between seemingly very different topics. Rene Bartsch later wrote to us: "It was a great pleasure for me to participate. In fact, I was surprised by the large amount of interesting things I’ve learned from the lectures, which you organized. It was much more than I would normally expect from a conference."